CERN-TH/2003-125
Bicocca-FT-03-17
hep-ph/0307035

SUDAKOV RESUMMATION OF MULTIPARTON QCD CROSS SECTIONS

Roberto BONCIANI[(a)] [*], **Stefano CATANI**[(b)], **Michelangelo L. MANGANO**[(c)] and **Paolo NASON**[(d)]

[(a)] Physikalisches Institut, Albert-Ludwig-Universität Freiburg, D-79104 Freiburg, Germany
[(b)] INFN, Sezione di Firenze, I-50019 Sesto Fiorentino, Florence, Italy
[(c)] CERN, Theoretical Physics Division, CH 1211 Geneva 23, Switzerland
[(d)] INFN, Sezione di Milano, I-20136 Milan, Italy

Abstract

We present the general expressions for the resummation, up to next-to-leading logarithmic accuracy, of Sudakov-type logarithms in processes with an arbirtrary number of hard-scattering partons. These results document the formulae used by the authors in several previous phenomenological studies. The resummation formulae presented here, which are valid for phase-space factorizable observables, determine the resummation correction in a process-independent fashion. All process dependence is encoded in the colour and flavour structure of the leading order and virtual one-loop amplitudes, and in Sudakov weights associated to the cross section kinematics. We explicitly illustrate the application to the case of Drell–Yan and prompt-photon production.

CERN-TH/2003-125
June 2003

[*]This work was supported by the European Union under contract HPRN-CT-2000-00149.

The perturbative QCD calculations of a large class of infrared and collinear safe observables are sensitive to Sudakov effects. Some classical examples of these observables are the e^+e^- energy–energy correlation in the back-to-back region [1], the cross section for Drell–Yan production of lepton pairs in hadron collisions [2], and several e^+e^- hadronic event shapes in nearly two-jet configurations [3].

The Sudakov effects appear when the observable is defined and/or measured close to the exclusive boundary of its phase space. We generically denote by y ($y > 0$) the kinematical variable that measures the distance from the exclusive boundary, so that the Sudakov region is specified by $y \ll 1$. When Sudakov sensitive observable are computed as power series expansions in the QCD coupling α_S, the perturbative series involves terms of the type $\alpha_S^n L^k$ ($k \leq 2n$), where $L = -\ln y$. These double logarithmic terms are due to final-state radiation of soft and collinear partons, and are a distinctive feature of any short-distance dynamics that is governed by an underlying gauge field theory. Since $L \gg 1$, the presence of logarithmically-enhanced terms spoils the convergence of the fixed-order expansion in $\alpha_S(Q^2)$, even if the observable is controlled by a typical hard-scattering scale Q whose value is large (such that $\alpha_S(Q^2) \ll 1$). The predictivity of perturbative QCD can be recovered by reorganizing the perturbative series according to the degree of divergence of the various logarithmic terms, and then by performing a systematic resummation, to all orders in α_S, of the contributions that are leading-logarithmic (LL), next-to-leading logarithmic (NLL), and so forth.

Resummed calculations up to NLL accuracy are available for several production cross sections in hadron collisions (see the list of references in Sect. 5 of Ref. [4]), and for many hadronic event shapes in e^+e^- annihilation (see e.g. Refs. [3, 5]) and in deep-inelastic lepton–hadron scattering (see e.g. Refs. [6]). The inclusion of resummed Sudakov effects increases the theoretical accuracy of perturbative calculations, by extending their applicability to wider phase-space regions and reducing the uncertainty coming from yet uncalculated higher-order terms. This brings about relevant improvements in phenomenological applications, as shown by the studies carried out in recent years [7]. For example, in e^+e^- annihilation the use of resummed calculations has become the standard procedure in the comparison with data on hadronic event shapes [8]: these calculations allow one to extend the perturbative treatment towards the two-jet region where statistics is higher; they also allow investigations of hadronic physics at the interface between perturbative and non-perturbative phenomena. In hadron collisions, resummed calculations often lead to a considerable reduction in the scale dependence of the perturbative predictions, as in the case of top quark production at the Tevatron and bottom quark production at HERA B [9, 10, 11].

In recent years, different groups (KLOS [12], BCMN [9, 10], BSZ [13]) have been working to develop resummation formalisms that are process-independent and observable-independent. The aim is to obtain generalized resummation formulae that depend on universal coefficients, and that are applicable to different hard-scattering processes and different classes of observables within the same process in terms of a minimal amount of information on the specific observable to be computed. We have explicitly checked that our generalized resummation formulae (which are presented here) reproduce known NLL results for several quantities, such as the thrust [14, 3] and C-parameter [15] distributions in e^+e^- annihilation, the cross sections for the production of lepton pairs, vector bosons [2] and Higgs bosons [16] in hadron collisions, the structure functions [17, 18] in deep-inelastic lepton–hadron scattering at large values of the Bjorken variable. We used this formalism to derive the NLL resummed results of Ref. [10] for the production of heavy quarks and prompt photons in hadron collisions. However, a general description of the formalism has never appeared in the literature. The purpose of this work is to fill this gap. Here we only give a brief illustration of our generalized resummation formulae. More details on the formalism and its derivation are given in a forthcoming paper.

The paper is organized as follows. We first consider QCD hard-scattering processes without hadrons in the initial state. We discuss the kinematic properties of the observables to which our resummation formalism applies. Then, we present our generalized resummation formula up to NLL accuracy. The explicit formula is expressed in terms of factorized final-state factors (J_i) and interference terms ($\mathbf{\Delta}^{(\mathrm{int.})}$). Then, we discuss the more general case of hard scattering in hadron collisions and in processes with tagged hadrons in the final state. Here the corresponding resummation formulae include additional initial- and final-state factors (Δ_i). We briefly illustrate the application of the general formalism by sketching the derivation of the resummation formulae presented in Ref. [10]. Finally, we summarize our main results.

We begin our presentation by considering a generic infrared- and collinear-safe cross section σ (or a related observable) in a hard process that does not involve hadrons in the initial state (for instance, hadron production in lepton collisions or in heavy-boson decays). We suppose that the calculation of σ at the leading order (LO) in QCD perturbation theory involves m final-state QCD partons with four-momenta $\{p_i\} = p_1, \dots, p_m$. For simplicity of presentation, we also limit ourselves to considering the case of massless ($p_i^2 = 0$) QCD partons (quarks, antiquarks and gluons). Using a shorthand notation, we write the LO contribution $\sigma^{(LO)}$ to the cross section as

$$\sigma^{(LO)} = \int \, \mathrm{d}\Phi(\sigma; \{p_i\}) \;\; |M^{(LO)}(\{p_i\})|^2 \; , \tag{1}$$

where $M^{(LO)}$ is the corresponding LO matrix element, and $|M^{(LO)}|^2$ denotes the squared matrix element summed over the colours and spins of the final-state QCD partons. The kinematics of the cross section are fully described by the phase-space factor $\mathrm{d}\Phi(\sigma; \{p_i\})$. It includes the phase-space contributions for the production of the final-state particles as well as any additional kinematics information (definition of jets, event shapes, energy flows, ...) that is necessary to precisely define the cross section σ that we want to evaluate. The phase-space dependence on σ is briefly indicated by the notation $\mathrm{d}\Phi(\sigma)$. In particular, $\mathrm{d}\Phi(\sigma)$ depends on the generic kinematic variable y that controls the distance from the Sudakov region. We assume that the LO term $\sigma^{(LO)}$ is well-behaved[†] (not singular) as $y \to 0$, while higher-order terms contain logarithmically-enhanced contributions of relative order $\alpha_S^n L^{2n}$. The dependence of σ on the momenta of non QCD partons $(\gamma, Z^0, W^\pm, H, \dots)$ is always understood.

Note that the Sudakov logarithms in σ do not necessarily occur by approaching the true physical phase-space boundary. These logarithms can also appear inside the phase space of certain observables[‡]. Indeed, logarithmically-enhanced terms may arise [19] also if the phase-space boundary for a certain number of partons lies inside that for a larger number, or if the observable itself is defined in a non-smooth way at some perturbative orders. In these cases, $\sigma^{(LO)}$ in Eq. (1) has to be regarded as the lowest-order contribution at which those partonic boundaries appear.

The practical feasibility of performing the resummation of the Sudakov logarithms at all perturbative orders depends on the capability of properly approximating the higher-order contributions to σ. The approximation regards both the QCD dynamics (i.e. the matrix elements) and the

[†]In practice, we consider the case in which all the LO invariants $p_i p_j$ are of the order of the hard scale Q^2 when $y \to 0$.

[‡]A notable example in e^+e^- annihilation is the C-parameter distribution, which has Sudakov logarithms in the vicinity of $C = 3/4$ [19]. Other examples are discussed, for instance, in Refs. [6, 20].

cross section kinematics. As for dynamics, since the Sudakov limit singles out multiple radiation of soft and collinear partons, we can exploit the universal (process-independent) factorization properties of the QCD multiparton matrix elements in the infrared (soft and collinear) region (see e.g. Refs. [21] and [22]). As for kinematics, we restrict our study to a (large) class of observables, whose phase space is *factorizable*. By phase-space factorization we precisely mean the following. At higher perturbative orders, we consider the contribution to σ from the final-state radiation of additional partons with momenta $q_1, \ldots, q_k$. In the Sudakov limit $y \to 0$, these momenta are kinematically forced to become soft or collinear to the momenta $\{p_i\}$, and the cross section σ is called factorizable if the corresponding phase space $\mathrm{d}\Phi(\sigma; p_1, \ldots, p_m, q_1, \ldots, q_k)$ behaves as

$$\mathrm{d}\Phi(\sigma; p_1, \ldots, p_m, q_1, \ldots, q_k) \underset{y \to 0}{\longrightarrow} \mathrm{d}\Phi(\sigma; \{p_i\}) \; [\mathrm{d}q] \; \prod_{j=1}^{k} u(\sigma, \{p_i\}; q_j) \;\; , \tag{2}$$

where $\mathrm{d}\Phi(\sigma; \{p_i\})$ is the LO phase space, $[\mathrm{d}q] = \prod_j \mathrm{d}^4 q_j \delta_+(q_j^2)/(2\pi)^3$ is the phase-space contribution from the unconstrained emission of the additional partons[§] with on-shell momenta $q_1, \ldots, q_k$, and on the right-hand side we have neglected relative corrections that vanish in the soft and collinear limit. The function $u(\sigma, \{p_i\}; q_j)$ is called Sudakov weight. It depends on the kinematical definition of the cross section σ (such dependence implicitly embodies the dependence on y), on the LO parton momenta $\{p_i\}$ and on a *single* (soft and collinear) final-state momentum q_j. The right-hand side of Eq. (2) implies that the kinematics dependence on the soft and collinear momenta is *fully* factorized: it is factorized with respect to the LO phase space and, moreover, there are no correlations between those momenta, since each Sudakov-weight factor depends on a single momentum q_j.

Note that the momenta $\{p_i\}$ on the right-hand side of Eq. (2) are not precisely the momenta of the LO partons on the left-hand side. The former exactly coincide with the latter in the soft limit $q_j \to 0$. When some of the momenta q_j are not soft but collinear to the momentum of one of the LO partons, say the parton i, the momentum p_i on the right-hand side is obtained by reabsorbing the longitudinal-momentum recoil produced by the collinear radiation.

As a consequence of the infrared and collineary safety of σ, the Sudakov weight fulfils the following important property:

$$u(\sigma, \{p_i\}; q) = 1 \qquad \text{when} \quad q = 0 \;, \quad \text{or} \quad q = (1-z)p_i \quad \text{for} \quad i = 1, \ldots, m \;\; . \tag{3}$$

Note that infrared and collinear safe observables are not necessarily factorizable. A classical example of non-factorizable observables are jet rates when the jets are defined by the JADE jet-finder algorithm [23]. Moreover, phase-space factorization is typically not achievable in the space of the kinematic variables where the cross section is originally defined. To overcome non-factorizable phase-space constraints, it is often necessary to introduce a conjugate space. For instance, the constraints of energy or transverse-momentum conservation are usually factorized by respectively performing Mellin (or Laplace) or Fourier transformations, and by working in the N-moment [2, 3, 24] or impact-parameter [1] space. Thus Eq. (2) can be valid either in the original space or in a properly defined conjugate space. In the following, y generically stands for either the original Sudakov variable or the variable conjugate to it (more precisely, the inverse of this conjugate variable) in the conjugate space.

§We are treating the partons as distinguishable particles. It the partons $j = 1, \ldots, k$ were identical, $[\mathrm{d}q]$ should be multiplied by a Bose-symmetry factor of $1/k!$.

To proceed further, we require one additional kinematics property on the observable to be resummed. In the Sudakov limit, the energy flow accompanying the LO hard scattering has to be suppressed *uniformly* with respect to its emission direction. To be precise, we consider the behaviour of the Sudakov weight $u(q)$ (from now on, $u(\sigma, \{p_i\}; q)$ is briefly denoted by $u(q)$) when $y \to 0$ at fixed value of q. We write the on-shell four-momentum $q^\mu = \omega(1, \hat{\mathbf{q}})$ in terms of its energy $q_0 = \omega$ and a three-dimensional vector $\hat{\mathbf{q}}$ of unit lenght ($\hat{\mathbf{q}}^2 = 1$), whose components parametrize the emission angle. When $y \to 0$, we thus require that $u(q) \to 0$ and, without loss of generality, we can always assume, to dominant logarithmic accuracy, that $u(q)$ is approximable by a step function:

$$u(q) \simeq \Theta\left(\omega_{\max} - \omega\right) \quad , \qquad \omega_{\max}(y; \{p_i\}, \hat{\mathbf{q}}) \underset{y \to 0}{\longrightarrow} 0 \quad , \tag{4}$$

where, the upper bound $\omega_{\max}$ on the radiated energy depends on the momenta $\{p_i\}$, on the emission angle $\hat{\mathbf{q}}$ and on the Sudakov variable y. In the Sudakov limit, we then require

$$\frac{\mathrm{d} \ln \omega_{\max}(y; \{p_i\}, \hat{\mathbf{q}})}{\mathrm{d} \ln y} \underset{y \to 0}{=} \lambda \quad , \tag{5}$$

where the power λ is *positive*, independent of y and $\{p_i\}$ and, in particular, *independent*[¶] of the radiation angle $\hat{\mathbf{q}}$. Equations (4) and (5) state in a formal way that, in the Sudakov limit, the parametric suppression rate of the energy flow emitted from the LO partons has to be uniform with respect to the radiation angle.

Note that, by requiring the property in Eqs. (4) and (5), we exclude from our resummation treatment observables such as away-from-jet energy flows, and, in general, the so-called non-global observables [25].

The general kinematic properties of phase-space factorization (see Eq. (2)) and uniform suppression of the energy flow (see Eqs. (4) and (5)) are sufficient to obtain our generalized resummation formula. The all-order cross section is generically denoted by σ_u. Here σ_u can be either the original cross section σ, or the corresponding cross section in the conjugate space where Eq. (2) applies (in this case, σ is eventually computed by perfoming the inverse transformation of σ_u to the original space). The cross section σ_u is written as

$$\sigma_u = \sigma_u^{(\mathrm{res})} + \sigma_u^{(\mathrm{fin})} \quad , \tag{6}$$

where the resummed component $\sigma_u^{(\mathrm{res})}$ contains all the Sudakov logarithms, while $\sigma_u^{(\mathrm{fin})}$ is well behaved (finite or vanishing) order by order in α_S when $y \to 0$. Thus, $\sigma_u^{(\mathrm{fin})}$ can reliably be evaluated by truncating its perturbative expansion at the first few perturbative orders. In practice, $\sigma_u^{(\mathrm{fin})}$ can be obtained from the fixed-order computation of σ_u by subtraction of the terms already included in $\sigma_u^{(\mathrm{res})}$ at the same fixed order.

The resummed component is given by

$$\sigma_u^{(\mathrm{res})} = \int \mathrm{d}\Phi_u(\sigma; \{p_i\}) \;\; |M^{(LO)}(\{p_i\})|^2 \;\; \Sigma(u) \; , \tag{7}$$

where $\mathrm{d}\Phi_u(\sigma; \{p_i\})$ is either the LO phase space $\mathrm{d}\Phi(\sigma; \{p_i\})$, or its version in the conjugate space. The expression (7) is completely analogous to the LO expression in Eq. (1). Sudakov resummation is simply achieved starting from the LO result and performing the replacement

[¶]To be precise, we allow Eqs. (4) and (5) to be violated in angular regions of vanishing solid angle (for instance, when q is exactly parallel to a LO momentum p_i).

$|M^{(LO)}|^2 \to |M^{(LO)}|^2\,\Sigma(u)$. The generalized effective form factor $\Sigma(u)$ embodies the dependence on the Sudakov logarithms to all perturbative orders. Since the Sudakov limit $y \to 0$ can formally be regarded as the limit $u \to 0$, the presence of logarithmically-enhanced terms in $\Sigma(u)$ is identified by contributions that order by order in α_S are formally divergent when $u(q) \to 0$.

The Sudakov logarithms exponentiate, and Σ has the following structure

$$\Sigma \sim C(\alpha_S) \exp\{\mathcal{G}(\alpha_S, L)\} = [1 + \alpha_S C_1 + \ldots]\ \exp\{L\, g_1(\alpha_S L) + g_2(\alpha_S L) + \ldots\} \quad . \tag{8}$$

The coefficient factor $C(\alpha_S)$ is independent of u (or y) and is due to hard virtual radiation. Its perturbative coefficients $C_1, C_2, \ldots$ depend on the specific cross section σ, but, since they are not logarithmically enhanced, they can be computed process by process at some finite perturbative orders. The exponent $\mathcal{G}(\alpha_S, L)$ contains the logarithmically-enhanced terms. The function $L\, g_1(\alpha_S L)$ resums the LL contributions $\alpha_S^n L^{n+1}$ in the exponent, the function $g_2(\alpha_S L)$ resums all the NLL contributions $\alpha_S^n L^n$, and so forth[‖].

The explicit NLL resummation formulae we are going to present have the structure of Eq. (8), with the only difference that the functions $C(\alpha_S)$ and $\mathcal{G}(\alpha_S, L)$ are matrices in the flavour and colour indices of the hard-scattering partons, so that exponentiation has to be understood in formal sense. The expression of the form factor Σ up to NLL accuracy is

$$\Sigma(u) = \left(\prod_{i=1}^{m} J_i(u) \right) \frac{\langle M_H(\{p_j\}) | \boldsymbol{\Delta}^{(\mathrm{int.})}(u) | M_H(\{p_j\}) \rangle}{|M^{(LO)}(\{p_j\})|^2} \quad . \tag{9}$$

The first factor on the right-hand side of Eq. (9) contains all the LL terms and part of the NLL terms. It is given by the product of the jet functions $J_i(u)$, and there is a jet function J_i for each final-state parton i in the corresponding LO process. The function $J_i(u)$, which generalizes the jet function of Refs. [2, 3, 10], embodies all the logarithmic terms produced by multiple radiation of partons that are collinear (either soft or not) to the direction of the momentum p_i of the LO parton i. The factorization in single-parton factors, J_i, is a consequence of the independent character (which follows from colour coherence) of QCD collinear radiation.

The remaining factor on the right-hand side of Eq. (9) contains NLL terms and subleading logarithmic contributions. The radiative factor $\boldsymbol{\Delta}^{(\mathrm{int.})}(u)$ generalizes the analogous radiative factor introduced in the case of prompt-photon hadroproduction [10]. It embodies all the quantum-interference effects produced by non-collinear (large-angle) soft-gluon radiation. In particular, this factor is sensitive to the colour correlations due to the colour flow dynamics of the LO hard scattering.

The function $J_i(u)$ has the following resummed expression:

$$\ln J_i(u) = 4\pi \int \frac{d^4 q}{(2\pi)^3}\, \delta_+(q^2)\, (u(q) - 1)\ \frac{\Theta(z_{iq})}{p_i q}\ \tilde{A}\left(\alpha_S(2(1 - z_{iq}) p_i q)\right)\ P_i(z_{iq}) \ , \tag{10}$$

where the functions $P_i(z)$, which depend on the flavour ($i = q, \bar{q}, g$) of the parton i, are related to the Altarelli–Parisi splitting functions

$$P_q(z) = P_{\bar{q}}(z) = C_F \frac{1 + z^2}{1 - z} \ , \quad P_g(z) = C_A \left[\frac{2}{1 - z} - 2 + z(1 - z) \right] + \frac{1}{2} N_f \left[z^2 + (1 - z)^2 \right] \ , \tag{11}$$

[‖] In our definition of LL, NLL, etc. terms, we are referring to the logarithmic hierarchy of the various contributions to the exponent $\mathcal{G}(\alpha_S, L)$ (i.e. to $\ln \Sigma$) in Eq. (8). Our systematic resummation procedure thus differs from the ones that refer to the expansion of Σ in successive logarithmic towers, such as $\alpha_S^n L^{2n}$, $\alpha_S^n L^{2n-1}$, etc.. In particular, our LL and NLL contributions include more logarithmic terms than those in the first two logarithmic towers of Σ (see, for instance, the discussion in Sect. 5 of Ref. [26]).

and $\tilde{A}(\alpha_S)$ is the QCD coupling as defined in the bremsstrahlung scheme [17], and is the related to the $\overline{\rm MS}$ coupling α_S by the NLO relation

$$\tilde{A}(\alpha_S) = \alpha_S \left[1 + \left(\frac{\alpha_S}{\pi} \right) \frac{K}{2} \right] \;, \quad K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \;. \tag{12}$$

The 'energy fraction' z_{iq} is defined with respect to a four-momentum n_i^μ as

$$z_{iq} = 1 - \frac{n_i q}{n_i p_i} \;. \tag{13}$$

Indeed, to obtain our resummed formulae, we have introduced m auxiliary momenta n_i^μ. These auxiliary momenta are arbitrary, with the only constraint of being *time-like* ($n_i^2 > 0$) and *hard*[**]. The LL terms in $J_i(u)$ do not depend on the definition of these auxiliary momenta. The NLL dependence of $J_i(u)$ on n_i is cancelled by the dependence on n_i in the remaining factor on the right-hand side of Eq. (9), so that our expression for $\Sigma(u)$ is independent of n_i up to corrections that are beyond the NLL accuracy of the present formalism. The main motivation for introducing the auxiliary momenta n_i^μ is to factorize collinear radiation in the jet functions $J_i(u)$ without the introduction of explicit angular boundaries. In practical calculations, the definition of the momenta n_i^μ can be adjusted to simplify the evaluation of the integral in Eq. (10).

The NLL contribution $\mathbf{\Delta}^{(\rm int.)}$ to the form factor is given by

$$\mathbf{\Delta}^{(\rm int.)}(u) = \overline{\mathbf{V}}(u) \; \mathbf{V}(u) \;\;, \tag{14}$$

where $\mathbf{V}(u)$ and $\overline{\mathbf{V}}(u)$ are matrices acting onto the colour indices of the LO partons. The explicit expression of $\mathbf{V}(u)$ is

$$\mathbf{V}(u) = P_z \; \exp \left\{ \sum_{i \neq j} \int_0^1 {\rm d}z \, \frac{\langle u(q) \rangle - 1}{1-z} \, \frac{\alpha_S((1-z)^2 \mu_R^2)}{4\pi} \; \sum_c T_i^c \, T_j^c \; \ln \frac{4 (n_i p_i)^2 (n_j p_j)^2}{(p_i p_j)^2 \, n_i^2 \, n_j^2} \right\} \;\;, \tag{15}$$

where μ_R is the renormalization scale, to be chosen of the order of the hard-scattering scale Q. The sum $\sum_{i \neq j}$ runs over the labels $i, j = 1, \dots, m$ of the LO partons and T_i^c, T_j^c ($c = 1, \dots, N_c^2 - 1$) are the corresponding colour charges: T_i^c is the colour matrix[††] in the fundamental (adjoint) representation if the parton i is a quark (gluon). The operator P_z denotes z-ordering in the formal expansion of the exponential matrix, and $\overline{\mathbf{V}}(u)$ is obtained from Eq. (15) by simply replacing P_z with $\overline{P}_z$, the ordering operator that acts in the opposite order. The notation $\langle u(q) \rangle$ in Eq. (15) stands for a properly defined average of the Sudakov weight $u(q)$. Parametrizing the light-like four-vector q^μ as $q^\mu = \omega(1, \hat{\mathbf{q}})$, the average is performed over the angular directions $\hat{\mathbf{q}}$ at fixed value $q_0 = \omega = (1-z)\mu_R$ of its energy q_0. In practice, exploiting Eqs. (4) and (5), and neglecting terms beyond NLL accuracy, we simply have

$$\langle u(q) \rangle = \Theta \left(y^\lambda - (1-z) \right) \;\;. \tag{16}$$

In Eq. (9), the NLL colour matrix $\mathbf{\Delta}^{(\rm int.)}$ acts onto the colour indices of the m-parton matrix element $M_H(\{p_i\})$. This hard matrix element is independent of u and is perturbatively computable as a power series in $\alpha_S(\mu_R^2)$:

$$M_H(\{p_i\}) = M^{(0)}(\{p_i\}) + \alpha_S(\mu_R^2) \; M_H^{(1)}(\{p_i\}) + \mathcal{O}(\alpha_S^2) \;\;, \tag{17}$$

[**]By hard, we mean that the invariants n_i^2 and $n_i p_j$ are of the order of the hard scale Q^2 when $y \to 0$.

[††]The colour charges are defined according to the notation in Sect. 3.2 of Ref. [27].

where $M^{(0)}$ is the LO matrix element and $M_H^{(1)}$ is the hard part of its one-loop virtual corrections. Since M_H does not containt logarithmically-enhanced terms, it can be evaluated by truncation of its perturbative expansion at some fixed order. In Eq. (9), the dependence of M_H on the colour indices is represented by the colour vectors $|M_H\rangle$ and $\langle M_H|$, which are defined according to the notation in Sect. 3.2 of Ref. [27].

The algebraic complications due to the non-trivial colour structure of the NLL contributions are straightforwardly overcomed when the number m of LO partons is $m = 2$ or $m = 3$. In these cases the colour algebra can be carried out in closed form, since the colour matrices $\sum_c T_i^c T_j^c$ in Eq. (15) are simply proportional to the unity matrix in colour space. The proportionality relations are (see Appendix A in Ref. [27])

$$\sum_c T_1^c T_2^c = -C_1 = -C_2 \qquad (m=2) \ , \tag{18}$$

$$\sum_c T_1^c T_2^c = \frac{1}{2}(C_3 - C_1 - C_2) \quad (m=3) \ , \tag{19}$$

where C_i is the Casimir invariant of the parton i ($C_i = C_A$ if the parton i is a gluon, $C_i = C_F$ if the parton i is a quark or an antiquark). When $m = 3$, $\sum_c T_2^c T_3^c$ and $\sum_c T_3^c T_1^c$ are obtained from Eq. (19) by permutation of the parton indices $\{1, 2, 3\}$. In the case of processes with $m \geq 4$ LO partons, the colour algebra cannot be handled without additional information on the hard matrix element $M_H(\{p_i\})$, since the colour can flow in many different ways through the hard scattering. In general, the evaluation of $\sum_c T_i^c T_j^c$ (and of Eq. (15)) requires the diagonalization of a linear combination of $m(m-3)/2$ independent colour matrices (see Appendix A in Ref. [27]).

The NLL resummed calculations of Ref. [12] deal with the non-trivial colour structure in the hard-scattering of $m = 4$ LO partons by considering 'soft anomalous dimensions' of gauge-dependent Wilson line operators. The Wilson line operators are introduced and properly defined on a process-dependent basis. In this respect, the integrand in the exponent of Eq. (15) can be regarded as a universal (process-independent and observable-independent) soft anomalous dimension matrix (in colour space), $\mathbf{\Gamma}$, of soft non-collinear gluons radiated by hard scattering of an arbitrary number ($m \geq 4$) of partons. The explicit expression of $\mathbf{\Gamma}$,

$$\alpha_S \, \mathbf{\Gamma}(\{p_i, n_i\}) = \alpha_S \sum_{i \neq j} \sum_c T_i^c T_j^c \, \ln \frac{4(n_i p_i)^2 (n_j p_j)^2}{(p_i p_j)^2 \, n_i^2 \, n_j^2} \ , \tag{20}$$

is gauge independent, though dependent on the auxiliary vectors n_i. The Altarelli–Parisi splitting function $P_i(z)$ in Eq. (10) controls the resummation of the Sudakov logarithms produced by collinear (soft and hard) radiation, and, by analogy, $\mathbf{\Gamma}$ controls the resummation of the Sudakov logarithms produced by soft non-collinear radiation. Obviously, the definition of the boundary between the collinear and non-collinear regions is quite arbitrary. In Eq. (20), this arbitrariness is somehow parametrized by the dependence on the auxiliary vectors n_i. For instance, using colour conservation, $\sum_{i=1}^m T_i^c = 0$ [27], Eq. (20) can be rewritten as

$$\alpha_S \, \mathbf{\Gamma}(\{p_i, n_i\}) = \alpha_S \sum_{i \neq j} \sum_c T_i^c T_j^c \, \ln \frac{4\mu_i^2 \mu_j^2}{(p_i p_j)^2} - 2\, \alpha_S \sum_{i=1}^m C_i \, \ln \frac{(n_i p_i)^2}{n_i^2 \, \mu_i^2} \ , \tag{21}$$

where μ_i are abitrary scales (e.g. $\mu_i = \mu_R$). The second term on the right-hand side is proportional to the unity matrix in colour space, and therefore it can be absorbed in a corresponding redefinition of the jet functions J_i in Eq. (9).

We now consider the general case of cross sections in hadron collision processes. In these processes, the hadronic cross section σ is obtained by convoluting the partonic cross sections σ_{a_1,a_2} with the parton densities $f_{a_1/h_1}(x_1,\mu_F^2)$ and $f_{a_2/h_2}(x_2,\mu_F^2)$ of the colliding hadrons with momenta P_1 and P_2:

$$\sigma = \sum_{a_1,a_2} \int_0^1 \mathrm{d}x_1 \, \mathrm{d}x_2 \; f_{a_1/h_1}(x_1,\mu_F^2) \; f_{a_2/h_2}(x_2,\mu_F^2) \; \sigma_{a_1,a_2} \;\; , \tag{22}$$

where μ_F is the factorization scale (to be chosen of the order of the hard-scattering scale Q), and the sum $\sum_{a_1,a_2}$ runs over the flavours ($a_1, a_2 = q, \bar{q}, g$) of the incoming partons with momenta $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$.

At the LO in QCD perturbation theory, the partonic cross section σ_{a_1,a_2} has the same structure as in Eq. (1), with m LO partons i ($i = 1,2$ are the incoming partons and $i = 3,\dots,m$ are the outgoing partons). The Sudakov logarithms at higher orders can be produced by soft and collinear radiation emitted from either the outgoing partons or the incoming partons in the LO hard scattering. In general, to kinematically factorize the Sudakov effects in the parton densities from those in the partonic cross section, it is necessary to consider the N-moments (Mellin moments) $f_{a/h,N}(\mu_F^2)$ of the parton densities,

$$f_{a/h,N}(\mu_F^2) = \int_0^1 \mathrm{d}x \; x^{N-1} \; f_{a/h}(x,\mu_F^2) \;\; , \tag{23}$$

and the corresponding N-moments $\sigma_{a_1,a_2}^{N_1,N_2}$ of the partonic cross section.

In the following we limit ourselves[‡‡] to considering the cases in which *hard* radiation collinear to the incoming partons is suppressed in the Sudakov limit, so that the Sudakov logarithms are produced by soft radiation (at any angles) and hard radiation collinear to the outgoing partons. This simplifies the presentation of the resummed formulae, since the Sudakov effects do not change the flavours a_1, a_2 of the incoming partons.

Our NLL resummed formulae apply to the N-moments $\sigma_{a_1,a_2}^{N_1,N_2}$ of the partonic cross sections that fulfil the factorization property of Eq. (2) in a properly defined conjugate space. We still require the properties in Eqs. (4) and (5). Infrared and collinear safety implies Eq. (3) in the case of radiation collinear to the outgoing partons $i = 3,\dots,m$. When $i = 1,2$ is an incoming parton, Eq. (3) is modified by a kinematical rescaling factor, which simply takes into account that we are considering the N_i-moments of the partonic cross section. We have

$$u(q) = z^{N_i-1} \simeq \exp\{-(N_i-1)(1-z)\} \qquad \text{when} \quad q = (1-z)p_i \quad \text{for} \quad i = 1,2 \;\; , \tag{24}$$

where the approximate equality is valid in the soft region ($1 - z \ll 1$) we are interested in. The all-order partonic cross section has the same structure as in Eq. (6), and its resummed component is obtained as in Eq. (7). The only difference is that the (final-state) form factor $\Sigma(u)$ has to be replaced by a more general radiative factor $\Delta_{a_1,N_1;a_2,N_2}(u)$. The latter is given by an expression similar to Eq. (9), apart from two simple modifications that regard the LL and NLL terms, respectively.

The modification of $\Sigma(u)$ at the LL level is obtained by performing the replacement

$$\left(\prod_{i=1}^{m} J_i(u) \right) \longrightarrow \Delta_{a_1,N_1}(u) \; \Delta_{a_2,N_2}(u) \; \left(\prod_{i=3}^{m} J_i(u) \right) \quad . \tag{25}$$

‡‡We thus exclude observables such as, for example, the Q_T-distribution of Drell–Yan lepton pairs, where powers of $\ln Q_T$ are produced also by radiation of hard quarks and gluons that are collinear to the colliding partons.

In other words, $\Delta_{a_1,N_1;a_2,N_2}(u)$ is obtained from $\Sigma(u)$ by supplementing the product of the final-state jet functions $J_i(u)$ with an initial-state factor $\Delta_{a_i,N_i}(u)$ for each incoming parton $i = 1, 2$. The initial-state Sudakov factor $\Delta_{N_i}(u)$, which generalizes the analogous N-moment factor of Refs. [2, 12, 10], resums the Sudakov logarithms produced by *soft* partons emitted collinearly to the LO incoming parton $i = 1, 2$. As in the case of the parton density $f_{a/h,N}(\mu_F^2)$, the radiative factor $\Delta_{N_i}(u)$ depends on the factorization scale μ_F and on the factorization scheme used to define the partonic cross section. The NLL resummed expression of $\Delta_{N_i}(u)$ in the $\overline{\rm MS}$ factorization scheme is

$$\begin{aligned} \ln \Delta_{a_i,N_i}(u) &= 4\pi \int \frac{{\rm d}^4 q}{(2\pi)^3}\, \delta_+(q^2)\, (u(q) - u_i(z_{iq}))\; \frac{\Theta(z_{iq})}{p_i q}\; \tilde{A}\left(\alpha_S(2(1-z_{iq})p_i q)\right)\; P_i(z_{iq}) \\ &+ \frac{C_i}{\pi} \int_0^1 {\rm d}z\; \frac{z^{N_i-1}-1}{1-z} \int_{\mu_F^2}^{4(1-z)^2(n_i p_i)^2/n_i^2} \frac{{\rm d}k^2}{k^2}\; \tilde{A}\left(\alpha_S(k^2)\right) \;\; . \end{aligned} \tag{26}$$

The term on the right-hand side of the first line of Eq. (26) is completely analogous to the right-hand side of Eq. (10) apart from the replacement $(u(q) - 1) \rightarrow (u(q) - u_i(z_{iq}))$, where $u_i(z) = u(q = (1-z)p_i)$. The Sudakov limit typically forces the parton distributions towards the large-N (large-x) region, and the term in Eq. (26) matches the sensitivity of the parton distribution $f_{a_i/h,N_i}(\mu_F^2)$ to large logarithms, $\ln N_i$, of the Mellin index N_i.

The modification of $\Sigma(u)$ at the NLL level regards the interference term $\mathbf{\Delta}^{(\rm int.)}$ or, more precisely, its components $\mathbf{V}(u)$ and $\overline{\mathbf{V}}(u)$ in Eq. (14). Equation (15) gives $\mathbf{V}(u)$ as an exponential of the soft anomalous dimension matrix $\mathbf{\Gamma}$ in Eq. (20). When going from processes with no initial-state partons to processes in hadron–hadron collisions, we have to take into account that the parton momenta p_1 and p_2 have to be crossed from the final to the initial state. As for the anomalous dimension $\mathbf{\Gamma}$, the crossing simply amounts to the following analytic continuation, $\ln(p_i p_j) \rightarrow \ln(p_i p_j) + i\pi$, of the terms with $i = 1$ or $i = 2$ and $j \geq 3$. Performing such a replacement, and using colour-charge conservation, $\sum_{j=3}^m T_i^c = -(T_1^c + T_2^c)$, we obtain the overall replacement to be applied to $\mathbf{\Gamma}$:

$$\mathbf{\Gamma}(\{p_i, n_i\}) \longrightarrow \mathbf{\Gamma}(\{p_i, n_i\}) \pm 4i\pi \sum_c (T_1^c + T_2^c)(T_1^c + T_2^c) \;\; . \tag{27}$$

Here, the signs $+$ and $-$ regard the replacements to be applied in the evaluation of $\mathbf{V}(u)$ and $\overline{\mathbf{V}}(u)$, respectively. Note that the substitution on the right-hand side of Eq. (27) is effective only in the case of hadron–hadron collisions with $m \geq 4$ hard-scattering partons at LO. In fact, when $m = 2$ we have $T_1^c + T_2^c = 0$, and when $m = 3$ we have $\sum_c (T_1^c + T_2^c)(T_1^c + T_2^c) = C_3^2$ so that the substitution in Eq. (27) changes $\mathbf{V}(u)$ by a pure phase factor that is cancelled by its complex conjugate phase factor in $\overline{\mathbf{V}}(u)$.

The replacement in Eq. (27), to be applied to the soft anomalous dimensions in hadron–hadron collisions, has a direct and interesting physical interpretation. As is well know from QED (see e.g. Ref. [28]), Coulomb-type virtual exchanges produce infrared-divergent Coulomb phases that affect any scattering amplitudes. Being them phases, they cancel in the evaluation of inclusive cross sections. The QCD analogue [22] of the QED Coulomb phases are non-abelian Coulomb 'phases', which are colour matrices. They lead to non-abelian infrared divergences that cancel, as in QED, in infrared- and collinear-safe observables with incoming massless partons [29, 22]. However, in the non-abelian case, the cancellation mechanism of the infrared divergences can produce residual (and non-trivial) finite contributions. The Sudakov logarithms produced by the $i\pi$-term on the right-hand side of Eq. (27) are a manifestation of these residual* finite effects. Very soft (and, hence, infrared divergent) non-abelian Coulomb-type interactions do cancel in σ. On the contrary, non-abelian Coulomb-type interactions of gluons that are harder (and, hence, infrared finite) than bremsstrahlung gluons do not cancel, since they are trapped by the colour fluctuations produced by the radiated bremsstrahlung gluons.

We note that our resummation formulae apply also to processes in which the partonic cross section has to be convoluted with the partonic fragmentation functions of hadrons that are tagged in the final state. The resummation formulae for these processes are simply obtained by multiplicatively introducing a factor of $\Delta_{a_i,N_i}(u)$ for each final-state parton i whose momentum has to be convoluted with the N_i moment of the corresponding fragmentation function. The fragmentation factor $\Delta_{a_i,N_i}(u)$ is the same as for parton densities (i.e. Eq. (26)), provided the fragmentation functions are defined in the $\overline{\rm MS}$ factorization scheme.

To illustrate the use of our generalized resummation formulae, we briefly sketch their application to the hadroproduction of Drell–Yan lepton pairs [2] and of prompt photons [10]. In both cases, the hadronic cross section is obtained by the factorization formula in Eq. (22), and $\sqrt{S}$ ($S = (P_1 + P_2)^2$) denotes the centre–of–mass energy. The hard-scattering scale is the invariant mass Q of the lepton pair in the first case, and the transverse energy E_T of the photon in the second case. The variable y that parametrizes the distance from the Sudakov region is respectively given by $y = 1 - Q^2/S$ and $y = 1 - 4E_T^2/S$. We are interested in the corresponding inclusive total (i.e. integrated over the rapidity of the observed final state) cross sections in the Sudakov limit $y \ll 1$. We thus consider the N-moments of the cross section in Eq. (22). The N-moments are defined with respect to $(1 - y)$ at fixed values of Q and E_T, respectively. This sets $N_1 = N_2 = N$ in the corresponding partonic cross sections $\sigma_{a_1,a_2}^{N_1,N_2}$. Since the moment variable N is conjugate to y through the Mellin transformation, the Sudakov limit corresponds to $1/N \to 0$ (i.e. $N \to \infty$). It is not difficult to prove that the factorization property of Eq. (2) is valid in N-space for both processes.

In the Drell–Yan process, the LO hard-scattering subprocess is $q(p_1) + \bar{q}(p_2) \to \ell\ell'(Q)$: the number of LO partons is $m = 2$ and the LO kinematics set $Q^2 = 2p_1p_2$. The Sudakov weight is

$$u_{DY}(q) = \exp\left\{ -N \frac{(p_1+p_2)q}{p_1p_2} \right\} \quad , \tag{28}$$

which fulfils Eqs. (4) and (5) with $\lambda = 1$. Since the LO process has no final-state partons, the Sudakov resummation factor $\Delta_{DY,N}$ is the product of $\boldsymbol{\Delta}^{(\rm int.)}$ and two initial-state factors, $\Delta_{q_1,N}$ and $\Delta_{\bar{q}_2,N}$. To apply our generalized resummation formulae, it is convenient to choose the auxiliary vectors as $n_1 = n_2 = p_1 + p_2$. We thus have $\boldsymbol{\Delta}^{(\rm int.)} = 1$ (see Eqs. (14) and (15)) and $\Delta_{q_1,N} = \Delta_{\bar{q}_2,N}$ (see Eq. (26)). Moreover, since $u(q) = u_i(z_{iq})$, the term on the right-hand side of the first line of Eq. (26) vanishes. Therefore, from the second line of Eq. (26) we finally obtain the known NLL result for the Drell–Yan process [2].

In the prompt-photon process (see the second paper in Ref. [10]), there are two independent LO hard-scattering subprocesses, $a_1(p_1) + a_2(p_2) \to a_3(p_3) + \gamma(p_\gamma)$, with $\{a_1 = q, a_2 = \bar{q}, a_3 = g\}$ and $\{a_1 = q, a_2 = g, a_3 = q\}$. The number of LO partons is $m = 3$ and, in the Sudakov limit the

*The $i\pi$-term in Eq. (27) is not the absolute overall effect of Coulomb-type interactions in hadron–hadron collisions. It is the relative effect produced by the Coulomb-phase mismatch between processes with no initial-states hadrons and processes in hadron–hadron collisions. Thus, it has conveniently been introduced by the corresponding analytic continuation procedure.

LO kinematics set $2E_T^2 \simeq 2p_1p_3 \simeq 2p_2p_3 \simeq p_1p_2$. The Sudakov weight is

$$u_\gamma(q) = \exp\left\{-N\frac{2p_3q}{p_1p_2}\right\} \quad , \tag{29}$$

which fulfils Eqs. (4) and (5) with $\lambda = 1$. The Sudakov resummation factor $\Delta_{a_1a_3\to a_3\gamma,N}$ is the product of two initial-state factors ($\Delta_{a_1,N}$ and $\Delta_{a_2,N}$), one final-state factor (J_{a_3}) and the NLL factor $\boldsymbol{\Delta}^{(\mathrm{int.})}$. To explicitly evaluate these factors, it is convenient to choose the following auxiliary vectors: $n_1 = p_1 + p_3$, $n_2 = p_2 + p_3$, $n_3 = p_1 + p_2 + p_3$. Using this choice, it is straightforward to check that the term on the right-hand side of the first line of Eq. (26) gives subleading (beyond the NLL accuracy) contributions. As for the colour algebra, we can simply use Eq. (19). Performing the phase-space integrals in Eqs. (10), (15) and using Eq. (26), we obtain the explicit resummed results anticipated in Sect. 4.2 of the the second paper in Ref. [10] and implemented in the phenomenological study of Ref. [30].

We have discussed a generalized formalism to perform the resummation of Sudakov logarithms in QCD hard-scattering processes. We have presented explicit resummation formulae up to NLL accuracy. The formulae are observable-independent and process-independent. The dependence on the observable is universally encoded in the one-particle Sudakov weight $u(\sigma, \{p_i\}; q)$. The dependence on the process is completely specified by flavour, colour charge and kinematics of the LO partons, the LO matrix element and its hard virtual corrections at one-loop order. This is the minimal amount of process-dependent information that is necessary in any calculations at fixed perturbative order. Within a specific process, the formalism is applicable to a large class of QCD observables that are specified by some kinematic properties, such as phase-space factorizability. Phase-space factorization has already been exploited in the literature to perfom resummation of several observables in processes controlled by LO hard scattering of two and three QCD partons. The extension of the NLL resummation techniques to multiparton processes requires a formalism able to deal with the radiation pattern of non-collinear soft gluons emitted by the colour flow dynamics of the underlying hard-scattering. Available NLL resummed calculations of some specific cross sections in four-parton processes treat the colour flow dynamics on a process-dependent basis. As for processes with higher number of LO partons, no NLL resummed calculation has been presented so far. Our NLL formalism applies to arbitrary processes with any number of hard-scattering partons and with arbitrary colour flow dynamics. This opens prospects of phenomenological applications to multijet events at present and future high-energy colliders.

References

[1] Y. L. Dokshitzer, D. Diakonov and S. I. Troian, Phys. Rept. **58** (1980) 269; G. Parisi and R. Petronzio, Nucl. Phys. B **154** (1979) 427; J. C. Collins and D. E. Soper, Nucl. Phys. B **193** (1981) 381 [Erratum-ibid. B **213** (1983) 545].

[2] G. Sterman, Nucl. Phys. B **281** (1987) 310; S. Catani and L. Trentadue, Nucl. Phys. B **327** (1989) 323, Nucl. Phys. B **353** (1991) 183.

[3] S. Catani, L. Trentadue, G. Turnock and B. R. Webber, Nucl. Phys. B **407** (1993) 3.

[4] S. Catani et al., hep-ph/0005025, in the Proceedings of the CERN Workshop on *Standard Model Physics (and more) at the LHC*, eds. G. Altarelli and M.L. Mangano (CERN 2000-04, Geneva, 2000), p. 1.

[5] S. Catani, G. Turnock and B. R. Webber, Phys. Lett. B **295** (1992) 269; Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, JHEP **9801** (1998) 011; A. Banfi, G. Marchesini, Y. L. Dokshitzer and G. Zanderighi, JHEP **0007** (2000) 002; A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, JHEP **0108** (2001) 047.

[6] V. Antonelli, M. Dasgupta and G. P. Salam, JHEP **0002** (2000) 001; M. Dasgupta and G. P. Salam, JHEP **0208** (2002) 032.

[7] See S. Frixione, Nucl. Phys. Proc. Suppl. **117** (2003) 222 and references therein.

[8] S. Bethke, J. Phys. G **26** (2000) R27.

[9] R. Bonciani, University of Florence Thesis, July 1997 (in italian).

[10] R. Bonciani, S. Catani, M. L. Mangano and P. Nason, Nucl. Phys. B **529** (1998) 424; S. Catani, M. L. Mangano and P. Nason, JHEP **9807** (1998) 024.

[11] M. Cacciari, S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, preprint CERN-TH-2003-054 (hep-ph/0303085).

[12] N. Kidonakis and G. Sterman, Phys. Lett. B **387** (1996) 867, Nucl. Phys. B **505** (1997) 321; N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. B **531** (1998) 365; E. Laenen, G. Oderda and G. Sterman, Phys. Lett. B **438** (1998) 173.

[13] A. Banfi, G. P. Salam and G. Zanderighi, JHEP **0201** (2002) 018, preprint LPTHE-03-12 (hep-ph/0304148).

[14] S. Catani, G. Turnock, B. R. Webber and L. Trentadue, Phys. Lett. B **263** (1991) 491.

[15] S. Catani and B. R. Webber, Phys. Lett. B **427** (1998) 377.

[16] S. Catani, D. de Florian and M. Grazzini, JHEP **0105** (2001) 025.

[17] S. Catani, B. R. Webber and G. Marchesini, Nucl. Phys. B **349** (1991) 635.

[18] H. Contopanagos, E. Laenen and G. Sterman, Nucl. Phys. B **484** (1997) 303.

[19] S. Catani and B. R. Webber, JHEP **9710** (1997) 005.

[20] P. Aurenche, M. Fontannaz, J. P. Guillet, A. Kotikov and E. Pilon, Phys. Rev. D **55** (1997) 1124; S. Catani, M. Fontannaz and E. Pilon, Phys. Rev. D **58** (1998) 094025; F. De Fazio and M. Neubert, JHEP **9906** (1999) 017; T. Binoth, J. P. Guillet, E. Pilon and M. Werlen, Eur. Phys. J. C **16** (2000) 311; A. K. Leibovich, I. Low and I. Z. Rothstein, Phys. Rev. D **62** (2000) 014010.

[21] A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rept. **100** (1983) 201; Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller and S. I. Troyan, *Basics of Perturbative QCD* (Editions Frontières, Gif-sur-Yvette, 1991).

[22] S. Catani and M. Ciafaloni, Nucl. Phys. B **249** (1985) 301; S. Catani, M. Ciafaloni and G. Marchesini, Nucl. Phys. B **264** (1986) 588.

[23] N. Brown and W. J. Stirling, Phys. Lett. B **252** (1990) 657.

[24] S. Catani, M. L. Mangano, P. Nason and L. Trentadue, Nucl. Phys. B **478** (1996) 273.

[25] M. Dasgupta and G. P. Salam, Phys. Lett. B **512** (2001) 323, JHEP **0203** (2002) 017; A. Banfi, G. Marchesini and G. Smye, JHEP **0208** (2002) 006; R. B. Appleby and M. H. Seymour, JHEP **0212** (2002) 063; C. F. Berger, T. Kucs and G. Sterman, preprint YITP-SB-03-06 (hep-ph/0303051); Y. L. Dokshitzer and G. Marchesini, JHEP **0303** (2003) 040.

[26] S. Catani, hep-ph/0005233, published in S. Catani *et al.*, hep-ph/0005114, summary report of the QCD/SM working group for the Workshop *Physics at TeV Colliders*, Les Houches, France, June 1999.

[27] S. Catani and M. H. Seymour, Nucl. Phys. B **485** (1997) 291 [Erratum-ibid. B **510** (1997) 503].

[28] S. Weinberg, *The Quantum Theory Of Fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, 1995).

[29] R. Doria, J. Frenkel and J. C. Taylor, Nucl. Phys. B **168** (1980) 93; C. Di'Lieto, S. Gendron, I. G. Halliday and C. T. Sachrajda, Nucl. Phys. B **183** (1981) 223.

[30] S. Catani, M. L. Mangano, P. Nason, C. Oleari and W. Vogelsang, JHEP **9903** (1999) 025.